

Paper Reference 9FM0/4A
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

PAPER 4A: Further Pure Mathematics 2

Monday 24 June 2024 – Afternoon

Time: 1 hour 30 minutes

**DO NOT RETURN THIS QUESTION PAPER
WITH THE ANSWER BOOKLET AT THE END OF
THE EXAMINATION.**

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

YOU MUST HAVE

**Mathematical Formulae and Statistical Tables (Green),
calculator**

YOU WILL BE GIVEN

**Diagram Booklet
Answer Booklet**

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

**There are 8 questions in this Question Paper.
The total mark for this paper is 75.**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. In this question you must show detailed reasoning.

Use Fermat's Little Theorem to determine the least positive residue of

$$21^{80} \pmod{23}$$

(Total for Question 1 is 4 marks)

2. Determine a closed form for the recurrence system

$$u_1 = 4 \quad u_2 = 6$$

$$u_{n+2} = 6u_{n+1} - 9u_n \quad (n = 1, 2, 3, \dots)$$

(Total for Question 2 is 5 marks)

3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (a) Use the Euclidean Algorithm to determine the highest common factor **h** of **234** and **96**

(3 marks)

- (b) Hence determine integers **a** and **b** such that

$$234a + 96b = h$$

(3 marks)

- (c) Solve the congruence equation

$$96x \equiv 36 \pmod{234}$$

(5 marks)

(Total for Question 3 is 11 marks)

4.

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 0 \\ 2 & p & -2 \\ 0 & -2 & 2 \end{pmatrix} \quad \text{where } p \text{ is a constant}$$

Given that

$$\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \text{ is an eigenvector of } \mathbf{A},$$

(a) determine the eigenvalue corresponding to this eigenvector.

(2 marks)

(b) Hence show that

$$p = 3$$

(1 mark)

(continued on the next page)

4. continued.

(c) Determine

(i) the remaining eigenvalues of **A**,

(ii) corresponding eigenvectors for these eigenvalues.

(6 marks)

(d) Hence determine a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^T$$

(3 marks)

(Total for Question 4 is 12 marks)

5. (i) A circle **C** in the complex plane is defined by the locus of points satisfying

$$|z - 3i| = 2|z|$$

- (a) Determine a Cartesian equation for **C**, giving your answer in simplest form.
(3 marks)
- (b) On an Argand diagram, shade the region defined by

$$\{z \in \mathbb{C} : |z - 3i| > 2|z|\}$$

(2 marks)

(continued on the next page)

5. continued.

(ii) The transformation T from the z -plane to the w -plane is given by

$$w = z^3$$

(a) Describe the geometric effect of T
(2 marks)

The region R in the z -plane is given by

$$\left\{ z \in \mathbb{C} : 0 < \arg z < \frac{\pi}{4} \right\}$$

(b) On a DIFFERENT Argand diagram, sketch the image of R under T
(2 marks)

(Total for Question 5 is 9 marks)

6. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$I_n = \int \frac{\cos(nx)}{\sin x} dx \quad n \geq 1$$

- (a) Show that, for $n \geq 1$

$$I_{n+2} = \frac{2 \cos(n+1)x}{n+1} + I_n$$

(6 marks)

(continued on the next page)

6. continued.

(b) Hence determine the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos(5x)}{\sin x} dx$$

giving the answer in the form

$a + b \ln c$ where a , b and c are rational numbers to be found.

(5 marks)

(Total for Question 6 is 11 marks)

7. The set of matrices

$\mathbf{G} = \{\mathbf{I}, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}\}$ where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

with the operation $\otimes_2$ of matrix multiplication with entries evaluated modulo 2, forms a group.

(a) Show that $\mathbf{B}$ is an element of order 3 in $\mathbf{G}$

(2 marks)

(b) Determine the orders of the other elements of $\mathbf{G}$

(3 marks)

(continued on the next page)

7. continued.

(c) Give a reason why **G** is NOT isomorphic to

(i) a cyclic group of order 6

(ii) the group of symmetries of a regular hexagon.

(2 marks)

The group **H** of permutations of the numbers 1, 2 and 3 contains the following elements, denoted in two-line notation,

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad a = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$c = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \quad d = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \quad f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

(d) Determine an isomorphism between the groups **G** and **H**

(3 marks)

(Total for Question 7 is 10 marks)

8. Refer to Diagram 1 and Diagram 2 for Question 8 in the Diagram Booklet.

Diagram 1 shows a French horn with a detachable bell section.

The shape of the bell section can be modelled by rotating an exponential curve through 360° about the X -axis, where units are centimetres.

The model uses the curve shown in Diagram 2, with equation

$$y = \frac{9}{2}e^{\frac{1}{9}x} \quad 0 \leq x \leq 9$$

(continued on the next page)

8. continued.

- (a) Show that, according to this model, the external surface area of the bell section is given by

$$K \int_0^9 e^{\frac{1}{9}x} \sqrt{4 + e^{\frac{2}{9}x}} \, dx$$

where **K** is a real constant to be determined.

(3 marks)

(continued on the next page)

8. continued.

(b) Refer to the information for Question 8(b) in the Diagram Booklet.

Use the substitution $u = e^{\frac{1}{9}x}$ to show the equation in the Diagram Booklet, where a and b are constants to be determined.

(5 marks)

Hence, using algebraic integration,

(c) determine, according to the model, the external surface area of the bell section of the horn, giving your answer to 3 significant figures.

(5 marks)

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
